The Mathematics of Mortality: Understanding Life Expectancy

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Paper prepared for

The Numbers Game – Money, Math & Beyond
Atlantic Provinces Trial Lawyers Association (APTLA) Conference
Halifax, Nova Scotia

November 21, 2014

Note: Readers who are constrained for time may wish to read the one-page article on this subject (originally published in the June 27, 2014 issue of The Lawyers' Weekly) which is appended to the end of this paper.

Mortality may be the most poorly understood of the actuarial assumptions in an economic loss valuation. Many lawyers are surprised to learn that actuaries do not use a plaintiff's life expectancy when valuing future lost earnings or care costs. Rather, they use mortality rates.

Confused? Let's start with some definitions and a couple of examples. We'll then wrap up with some concrete advice concerning how to obtain and interpret an expert mortality opinion.

Life Expectancy

> Life expectancy is the average future lifetime of a group of individuals aged x (actuaries like it when the plaintiff is x years old!) according to a specific mortality table. According to the 2009-2011 Life Tables for Canada, the life expectancy of a 45-year-old man is 36.17 years or to age 81.17. Some of these 45-year-olds will die tomorrow or next week. Others will live into their 90s or beyond. The average future lifetime of this diverse group is 36.17 years.
>
> Life expectancy is a useful concept when comparing two different mortality tables or when comparing a healthy member of the general population to someone with significant health issues. However, life expectancies are not used in actuarial calculations.

Mortality Rates and Mortality Tables

A mortality rate is the probability of death in a given time interval, according to a specific mortality table. A mortality table is a series of year-by-year mortality rates that are based on the study of mortality experience in a given population (for example: all of Canada, a single province, the policy-holders of one insurance company, etc.).

The 2009-2011 Life Table for Canada is the mortality table most commonly used by economic loss experts today. It is published by Statistics Canada and represents the most up-to-date measurement of mortality experience in the general Canadian population. The 2009-2011 Life Table starts at birth and ends at age 110 (according to the table, nobody lives beyond this age). Here's an excerpt from the male table:

Age	Mortality Rate (Probability of Death)
45	0.00194
46	0.00211
47	0.00229
48	0.00251

According to this table, if you survive until age 47, you have a 0.229% chance of dying before your 48th birthday. Translated to English, that's slightly more than a 2 in 1,000 chance of dying between your 47th and 48th birthdays.

The full 2009-2011 Life Table for Canadian males can be found in Appendix A to this paper. It is the probabilities of survival (p_x column) and death (q_x column) that actuaries use in their economic loss calculations. This use of probabilities is known as the Actuarial Present Value Method.

Actuarial Present Value Method

Under this method, which has long been accepted by the courts, the loss in each future year is discounted by the likelihood of survival to that year based on the age-by-age mortality rates of the accepted table. For a lifetime loss, this calculation is applied separately to each future year up to age 100 and beyond, according to the limits of the accepted table. The interest discount factors are similarly applied on a year-by-year basis. It is the Actuarial Present Value Method which accurately determines the lump-sum amount that will be exactly sufficient, on average, (no more and no less) to replace the stipulated loss.

For example, if there will be a care cost outlay of $1,000 one year from now and the plaintiff has a 10% chance of dying in the next year (therefore a 90% probability of living through the year), then the expected outlay is $900 (90% of $1,000). This $900 is the amount that will be discounted, by one year of interest in this particular example, to the valuation date when the Actuarial Present Value Method is used.

Substandard Mortality

An individual has substandard mortality if their year-by-year probability of death is higher than the standard mortality rates of the mortality table being used. Actuaries will sometimes use the term "shortened life expectancy" instead of "substandard mortality" to assist the reader in picturing what is being communicated.

A Personal Injury Example – Substandard Mortality

If a medical expert determines that a 45-year-old male plaintiff has substandard mortality and is 4 times more likely to die each future year than the average Canadian male, then this individual has a 0.776% probability of dying prior to his 46th birthday (4 times the standard mortality rate of 0.194% shown above), a 0.844% chance of dying prior to his 47th birthday (4 x 0.211%) if he first survives until his 46th birthday, etc. This individual's life expectancy is 23.30 years or to age 68.30.

As mentioned earlier, it's the actual mortality rates (either standard or substandard) that are used in the economic loss valuation. If there will be a care cost outlay of $1,000 one year from now and the plaintiff has a 10% chance of dying in the next year (therefore a 90% probability of living through the year), then the expected outlay is $900 (90% of $1,000). This $900 is the amount that will be discounted with interest to the valuation date.

Where substandard mortality is proven, the quantum of damages will be reduced. Here are some examples for our sample 45-year-old male*:

Mortality Assumption	Value of Future Lost Earning Capacity ($50,000 per annum, retirement at age 65)	Value of Future Care Costs ($10,000 per annum, for life)	Life Expectancy (years/to age)
Standard	$763,400	$231,800	36.17 / 81.17
2 x Standard	$739,000	$202,400	29.55 / 74.55
3 x Standard	$715,800	$183,900	25.84 / 70.84
4 x Standard	$693,800	$170,300	23.30 / 68.30
5 x Standard	$672,900	$159,500	21.39 / 66.39

* 2009-2011 Life Tables for Canada and prescribed interest discount rate of 2.5% for New Brunswick, Nova Scotia (non-MVA), and PEI

What does the above table tell us?

- Substandard mortality has a greater impact on a lifetime loss (such as future care costs) than on a temporary loss (such as future earning capacity to a specified retirement age). If it is proven that the 45-year-old plaintiff has a 5 times higher than normal probability of dying each year, the value of future lost earning capacity decreases by about 10% and the value of future care costs decreases by about 30%.

- Substandard mortality often has less of an impact on the value of the pecuniary damages than might be expected. Although a "5 times higher than normal probability of dying each year" seems quite significant, for a middle-aged male this increases one's annual probability of death from around 2 in 1,000 to about 10 in 1,000. Even with significantly substandard mortality, one's chances of surviving a given year remain much greater than one's chances of dying during that year.

- If the plaintiff was of average health prior to the injury and the substandard mortality is solely the result of the injury, then the value of future lost earnings would increase – all other things being equal – since the value of the residual earnings is decreased due to the substandard mortality.

Are you ready for another definition?

Limiting Age

The limiting age is the age at which an annual loss is assumed to end, even if the plaintiff survives to beyond that age. In personal injury matters, the most common limiting ages are:

- Retirement age – for future lost earning capacity, and
- The age (usually 70, 75, or 80) at which it is assumed the plaintiff would not have been able to perform valuable household services even if the accident that gave rise to the injury had not occurred

Mistaking Life Expectancy for Limiting Age

Two types of errors can arise if the life expectancy age (average future lifetime) is assumed to be the same as the limiting age (maximum age):

- If you assume that the plaintiff will live (with certainty) until the end of their life expectancy and then will die, then it can be proven mathematically that you will always overvalue the loss when compared to the proper actuarial present value method. While this imbedded error may be appealing from the plaintiff lawyer perspective, this computational error won't seem as attractive when the actuary on the other side discredits your expert by pointing out the error.

- Conversely, if you use the proper mortality rates and also assume that the plaintiff will die (with certainty) at the end of their life expectancy, then you are certain to undervalue the loss.

A Real Estate Example

A real estate example may help to illustrate both how "life expectancy age" differs from "limiting age", and also the nature of the "undervaluing" error described above.

According to the Canada Housing and Mortgage Corporation (CMHC), the average house price in Halifax in 2012 was $270,742. In the same year, the average house price in Moncton was $158,107. These averages in two different cities are the real estate equivalent of life expectancies for two different mortality tables.

These average house prices give us a high-level picture of the relative cost of housing in the two cities. Life expectancy (average future lifetime) serves the same purpose when we wish to compare the expected longevity of different groups of people.

Let's move on to the "limiting age" concept. Say, for example, that the Halifax average house price was based on the following (fictitious) sales data for ten houses:

$185,000	$240,600
$192,360	$257,490
$195,650	$300,120
$206,000	$386,980
$220,080	$523,140

You can add up these ten house prices and divide the total by ten to confirm that the average house price is $270,742.

However, if you omit the three house prices that are greater than the average, you will arrive at a new, different (and incorrect) average Halifax house price of $213,883:

($185,000 + $192,360 + $195,650 + $206,000 + $220,080 + $240,600 + $257,490) / 7 = $213,883

If an actuary were to use the proper mortality rates and also assume that the plaintiff will die (with certainty) at the end of their life expectancy (i.e., use the life expectancy age as the limiting age), then that would be like re-calculating the average house price in Halifax by leaving out all of the houses that are more expensive than the average!

Health-adjusted Life Expectancy

Some life care planners introduce the concept of Health-adjusted Life Expectancy (HALE) into their reports. HALE takes into account both the probability of death and the risk of disability. Depending on the disability tables used, it can be a measure of average future years during which the plaintiff is expected to be able to work or it may instead be a measure of quality of life.

HALE is a useful concept. However, the economic loss expert cannot incorporate health-adjusted life expectancy directly into the valuation any more than the expert can incorporate regular life expectancy. For example, if HALE is used as the "limiting age" for the loss, the result is an undervaluation (see above).

Although the HALE cannot be used directly in the actuarial valuation, it can be used to determine an equivalent limiting age (for lost valuable services, for example). For a 45-year-old male plaintiff, following are some representative limiting ages based on the 2009-2011 Life Table for Canada, the age-specific disability incidence rates that were used for the 26th actuarial valuation of the Canada Pension Plan (December 31, 2012), and sample life care planner HALE opinions:

HALE Specified by the Life Care Planner	Equivalent Limiting Age Calculated by the Actuary
20 years, or to age 65	67.2
22 years, or to age 67	69.9
25 years, or to age 70	74.3

For example, if the life care planner specifies a HALE of 22 years (to age 67) for lost valuable services, a limiting age of 69.9 is required in order to properly reflect the life care planner's opinion as to the duration of the loss.

What Will the Expert Mortality Opinion Look Like?

The doctors and others who are accustomed to providing expert opinions for the courts will usually provide their opinion in the form of a recommended mortality assumption. For example, they may state that the plaintiff has increased mortality of +300%, which means 400% (100% + 300%) of standard mortality rates.

Other experts will create a customized mortality table that is specific to the plaintiff's situation. Their reports typically include a table with columns showing the year-by-year probabilities of survival or death that, in their expert opinion, are appropriate for the plaintiff.

The actuary can readily utilize either of the above two forms of opinion, without difficulty.

Other doctors will opine on life expectancy. When this occurs, your economic loss expert will have to work backwards from the life expectancy to determine the equivalent underlying mortality rate assumption. If the doctor opines that a 45-year-old male plaintiff has a life expectancy of 20 years, then the actuary will be able to determine that this is equivalent to the plaintiff having 465% of the standard year-by-year probabilities of death in the 2009-2011 Life Table for Canada. To determine the value of the economic loss, the actuary will use the substandard mortality assumption that yields exactly the life expectancy that was specified by the doctor.

A final group of medical experts will state that the plaintiff has a specified percentage chance of surviving for a given number of years (say, a 50% chance of surviving for 5 years). It can be very difficult to convert this type of medical opinion into a year-by-year mortality rate assumption. If the expert medical opinion takes this form, the actuary may have to obtain additional information or clarification from the medical expert.

Considerations When Retaining a Medical Expert

- Ask your medical expert to opine on the "mortality assumption" that is appropriate for the plaintiff. Do <u>not</u> ask for a "life expectancy" opinion. Physicians are not educated in the mathematics of life contingencies, which includes the calculation of life expectancy. The doctor's opinion on the increased risk of mortality to the plaintiff, as compared to the general population, is what's needed and is better aligned with the physician's expertise.

- If the medical expert is a treating physician, all of the relevant medical information will likely already be in the file. If you retain an independent mortality expert, then that expert will require relevant information (clinical notes, test results, diagnoses, etc.) from the treating physician's file.

- Most lawyers prefer to retain the medical expert directly, thereby controlling whether and how information is communicated from one expert to another. However, some lawyers choose to delegate this responsibility to the actuary since it is the actuary who will be using the medical opinion in preparing the pecuniary damage valuation.

- Depending on the complexity of the file and the type of medical expert retained, the fee for an expert medical opinion can range from under $1,000 to over $5,000.

- If you wish to obtain the mortality opinion from an independent expert rather than from a treating physician, your economic loss expert can assist you in identifying one or more appropriately qualified experts.

Final Words

Remember that life expectancy is <u>average</u> future lifetime, not <u>maximum</u> future lifetime (look back at that real estate example!). Don't be surprised or concerned if the multiplier tables in the valuation report continue past the stated life expectancy.

The personal injury lawyer needn't be an expert in the probability and statistics of mortality (a field known as life contingencies). However, you'll want to ensure that your economic loss expert is an expert in this facet of valuation – particularly if the plaintiff isn't of normal health.

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Appendix A – Life Tables for Canada, 2009-2011, Males

(published by Statistics Canada)

Table 1a
Complete life table, males, Canada, 2009 to 2011

Age	l_x	d_x	q_x	m.e.(q_x)	p_x	L_x	T_x	e_x	m.e.(e_x)
	number		probability			number		year	
0 year	100,000	522	0.00522	0.00018	0.99478	99,531	7,933,442	79.33	0.04
1 year	99,478	30	0.00030	0.00004	0.99970	99,463	7,833,911	78.75	0.04
2 years	99,449	22	0.00022	0.00004	0.99978	99,438	7,734,448	77.77	0.04
3 years	99,427	16	0.00017	0.00003	0.99983	99,418	7,635,010	76.79	0.04
4 years	99,411	13	0.00013	0.00003	0.99987	99,405	7,535,591	75.80	0.04
5 years	99,398	11	0.00011	0.00003	0.99989	99,392	7,436,187	74.81	0.04
6 years	99,387	10	0.00010	0.00003	0.99990	99,382	7,336,795	73.82	0.04
7 years	99,377	9	0.00009	0.00002	0.99991	99,373	7,237,413	72.83	0.04
8 years	99,369	8	0.00008	0.00002	0.99992	99,364	7,138,040	71.83	0.04
9 years	99,360	8	0.00008	0.00002	0.99992	99,356	7,038,675	70.84	0.04
10 years	99,352	9	0.00009	0.00002	0.99991	99,348	6,939,319	69.85	0.04
11 years	99,343	10	0.00010	0.00003	0.99990	99,339	6,839,971	68.85	0.04
12 years	99,334	12	0.00012	0.00003	0.99988	99,328	6,740,633	67.86	0.04
13 years	99,322	15	0.00015	0.00003	0.99985	99,315	6,641,305	66.87	0.04
14 years	99,308	20	0.00020	0.00003	0.99980	99,298	6,541,990	65.88	0.04
15 years	99,288	28	0.00028	0.00004	0.99972	99,274	6,442,692	64.89	0.04
16 years	99,260	39	0.00039	0.00005	0.99961	99,241	6,343,418	63.91	0.04
17 years	99,221	50	0.00051	0.00005	0.99949	99,196	6,244,177	62.93	0.04
18 years	99,171	59	0.00059	0.00006	0.99941	99,141	6,144,982	61.96	0.04
19 years	99,112	65	0.00066	0.00006	0.99934	99,079	6,045,840	61.00	0.04
20 years	99,047	70	0.00071	0.00006	0.99929	99,011	5,946,761	60.04	0.04
21 years	98,976	74	0.00075	0.00006	0.99925	98,939	5,847,750	59.08	0.04
22 years	98,902	76	0.00076	0.00006	0.99924	98,864	5,748,810	58.13	0.04
23 years	98,827	75	0.00076	0.00006	0.99924	98,789	5,649,946	57.17	0.04
24 years	98,752	73	0.00074	0.00006	0.99926	98,715	5,551,157	56.21	0.04
25 years	98,679	70	0.00071	0.00006	0.99929	98,644	5,452,442	55.25	0.04
26 years	98,609	69	0.00070	0.00006	0.99930	98,574	5,353,798	54.29	0.04
27 years	98,540	68	0.00069	0.00006	0.99931	98,506	5,255,223	53.33	0.04
28 years	98,472	69	0.00070	0.00006	0.99930	98,438	5,156,717	52.37	0.03
29 years	98,404	70	0.00071	0.00006	0.99929	98,369	5,058,279	51.40	0.03
30 years	98,333	73	0.00074	0.00006	0.99926	98,297	4,959,911	50.44	0.03
31 years	98,261	76	0.00078	0.00007	0.99922	98,223	4,861,614	49.48	0.03
32 years	98,184	80	0.00082	0.00007	0.99918	98,144	4,763,391	48.51	0.03
33 years	98,104	84	0.00086	0.00007	0.99914	98,062	4,665,247	47.55	0.03
34 years	98,020	89	0.00091	0.00007	0.99909	97,976	4,567,184	46.59	0.03
35 years	97,931	94	0.00096	0.00007	0.99904	97,884	4,469,209	45.64	0.03
36 years	97,837	100	0.00102	0.00008	0.99898	97,788	4,371,324	44.68	0.03
37 years	97,738	106	0.00108	0.00008	0.99892	97,685	4,273,537	43.72	0.03
38 years	97,632	113	0.00115	0.00008	0.99885	97,576	4,175,852	42.77	0.03
39 years	97,519	120	0.00123	0.00008	0.99877	97,459	4,078,276	41.82	0.03
40 years	97,399	129	0.00132	0.00008	0.99868	97,335	3,980,817	40.87	0.03
41 years	97,270	138	0.00142	0.00009	0.99858	97,201	3,883,482	39.92	0.03
42 years	97,132	148	0.00153	0.00009	0.99847	97,058	3,786,281	38.98	0.03
43 years	96,984	160	0.00165	0.00009	0.99835	96,904	3,689,223	38.04	0.03
44 years	96,824	173	0.00179	0.00009	0.99821	96,737	3,592,320	37.10	0.03
45 years	96,651	187	0.00194	0.00010	0.99806	96,557	3,495,582	36.17	0.03
46 years	96,464	203	0.00211	0.00010	0.99789	96,362	3,399,025	35.24	0.03
47 years	96,261	221	0.00229	0.00010	0.99771	96,150	3,302,663	34.31	0.03
48 years	96,040	241	0.00251	0.00011	0.99749	95,919	3,206,513	33.39	0.03
49 years	95,799	263	0.00275	0.00011	0.99725	95,667	3,110,594	32.47	0.03
50 years	95,536	288	0.00301	0.00012	0.99699	95,392	3,014,926	31.56	0.03
51 years	95,248	316	0.00331	0.00013	0.99669	95,090	2,919,535	30.65	0.03
52 years	94,932	346	0.00364	0.00013	0.99636	94,759	2,824,445	29.75	0.03
53 years	94,586	379	0.00401	0.00014	0.99599	94,397	2,729,685	28.86	0.03
54 years	94,207	415	0.00441	0.00015	0.99559	94,000	2,635,288	27.97	0.03

Table 1a
Complete life table, males, Canada, 2009 to 2011 (concluded)

Age	l_x	d_x	q_x	m.e.(q_x)	p_x	L_x	T_x	e_x	m.e.(e_x)
	number		probability			number		year	
55 years	93,792	454	0.00484	0.00016	0.99516	93,565	2,541,289	27.09	0.03
56 years	93,338	497	0.00533	0.00017	0.99467	93,089	2,447,723	26.22	0.03
57 years	92,841	544	0.00586	0.00018	0.99414	92,569	2,354,634	25.36	0.03
58 years	92,297	595	0.00645	0.00019	0.99355	91,999	2,262,065	24.51	0.03
59 years	91,701	650	0.00709	0.00021	0.99291	91,376	2,170,066	23.66	0.03
60 years	91,051	711	0.00780	0.00022	0.99220	90,696	2,078,690	22.83	0.03
61 years	90,340	776	0.00859	0.00023	0.99141	89,952	1,987,995	22.01	0.03
62 years	89,565	846	0.00945	0.00025	0.99055	89,141	1,898,042	21.19	0.03
63 years	88,718	923	0.01040	0.00026	0.98960	88,257	1,808,901	20.39	0.03
64 years	87,795	1,005	0.01145	0.00029	0.98855	87,293	1,720,644	19.60	0.03
65 years	86,790	1,094	0.01260	0.00032	0.98740	86,243	1,633,351	18.82	0.03
66 years	85,696	1,189	0.01387	0.00034	0.98613	85,102	1,547,108	18.05	0.03
67 years	84,507	1,291	0.01528	0.00037	0.98472	83,862	1,462,006	17.30	0.03
68 years	83,217	1,400	0.01682	0.00040	0.98318	82,517	1,378,144	16.56	0.03
69 years	81,817	1,515	0.01852	0.00043	0.98148	81,059	1,295,627	15.84	0.03
70 years	80,301	1,638	0.02040	0.00047	0.97960	79,482	1,214,568	15.13	0.03
71 years	78,663	1,767	0.02247	0.00050	0.97753	77,780	1,135,086	14.43	0.03
72 years	76,896	1,903	0.02475	0.00054	0.97525	75,944	1,057,306	13.75	0.03
73 years	74,993	2,045	0.02726	0.00058	0.97274	73,971	981,362	13.09	0.03
74 years	72,948	2,191	0.03004	0.00063	0.96996	71,853	907,391	12.44	0.03
75 years	70,757	2,342	0.03310	0.00067	0.96690	69,586	835,539	11.81	0.03
76 years	68,415	2,495	0.03647	0.00072	0.96353	67,167	765,953	11.20	0.03
77 years	65,920	2,650	0.04019	0.00076	0.95981	64,595	698,785	10.60	0.03
78 years	63,270	2,803	0.04430	0.00082	0.95570	61,869	634,190	10.02	0.03
79 years	60,467	2,953	0.04883	0.00088	0.95117	58,991	572,321	9.46	0.03
80 years	57,515	3,096	0.05383	0.00097	0.94617	55,967	513,330	8.93	0.03
81 years	54,419	3,230	0.05935	0.00105	0.94065	52,804	457,364	8.40	0.03
82 years	51,189	3,349	0.06543	0.00115	0.93457	49,514	404,560	7.90	0.03
83 years	47,840	3,452	0.07215	0.00126	0.92785	46,114	355,045	7.42	0.03
84 years	44,388	3,532	0.07957	0.00138	0.92043	42,622	308,931	6.96	0.03
85 years	40,856	3,585	0.08776	0.00153	0.91224	39,063	266,309	6.52	0.03
86 years	37,271	3,608	0.09680	0.00171	0.90320	35,467	227,246	6.10	0.03
87 years	33,663	3,594	0.10678	0.00193	0.89322	31,866	191,779	5.70	0.03
88 years	30,068	3,542	0.11780	0.00218	0.88220	28,297	159,914	5.32	0.03
89 years	26,526	3,448	0.12997	0.00255	0.87003	24,803	131,616	4.96	0.03
90 years	23,079	3,310	0.14341	0.00300	0.85659	21,424	106,813	4.63	0.04
91 years	19,769	3,122	0.15794	0.00355	0.84206	18,208	85,389	4.32	0.04
92 years	16,647	2,884	0.17326	0.00425	0.82674	15,205	67,182	4.04	0.04
93 years	13,763	2,605	0.18931	0.00503	0.81069	12,460	51,977	3.78	0.04
94 years	11,157	2,299	0.20604	0.00601	0.79396	10,008	39,517	3.54	0.05
95 years	8,858	1,935	0.21839	0.00706	0.78161	7,891	29,509	3.33	0.05
96 years	6,924	1,630	0.23536	0.00842	0.76464	6,109	21,618	3.12	0.06
97 years	5,294	1,339	0.25290	0.01039	0.74710	4,625	15,509	2.93	0.07
98 years	3,955	1,072	0.27092	0.01314	0.72908	3,420	10,884	2.75	0.08
99 years	2,884	834	0.28933	0.01658	0.71067	2,467	7,465	2.59	0.09
100 years	2,049	631	0.30802	0.01796	0.69198	1,734	4,998	2.44	0.11
101 years	1,418	464	0.32687	0.02328	0.67313	1,186	3,264	2.30	0.14
102 years	955	330	0.34576	0.03495	0.65424	790	2,078	2.18	0.19
103 years	625	228	0.36457	0.04540	0.63543	511	1,288	2.06	0.25
104 years	397	152	0.38319	0.06539	0.61681	321	778	1.96	0.35
105 years	245	98	0.40149	0.11196	0.59851	196	457	1.87	0.51
106 years	147	61	0.41937	0.11706	0.58063	116	261	1.78	0.74
107 years	85	37	0.43673	0.49621	0.56327	66	145	1.71	1.20
108 years	48	22	0.45350	0.38743	0.54650	37	79	1.64	0.98
109 years	26	12	0.46960	0.49403	0.53040	20	42	1.59	1.02
110 years and over	14	14	1.00000	0.00000	0.00000	22	22	1.56	…

Source: Statistics Canada, Demography Division.

Appendix B – Understanding the Mathematics of Mortality

**(article originally published by Lexis Nexis
in The Lawyers' Weekly, June 27, 2014)**

Focus PERSONAL INJURY

Understanding the mathematics of mortality


Jay Jeffery
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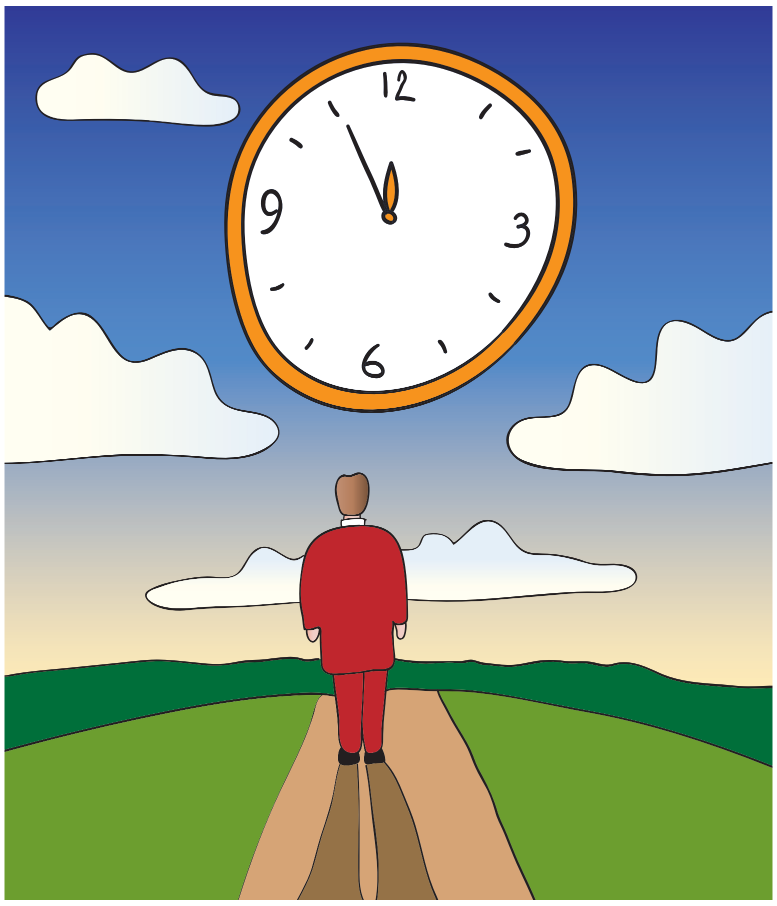
The concepts of mortality and life expectancy may be the most misunderstood of the actuarial assumptions used in economic loss valuations. Explained simply, life expectancy is the average future number of years of life remaining for a group of individuals at a particular age. It is a useful concept for visualization and for comparing different mortality tables or assumptions.

Many lawyers are surprised to learn that a plaintiff's "life expectancy" is not actually used in a proper present value calculation. Rather, the courts have long accepted that the correct allowance for the mortality contingency is based on the actuarial present value method. Under this method, the loss in each future year is discounted by the likelihood of survival to that year based on the age-by-age mortality rates of the accepted statistical table. For a lifetime loss, this calculation is applied separately to each future year up to age 100 and beyond, according to the limits of the accepted table. It is this method which accurately determines the amount required to be exactly sufficient on average (no more and no less) to replace the stipulated loss.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Confused by the difference? Let's look at an example.

Suppose we wish to determine the present value of a future loss for a male aged 45 in the amount of $1,000 per year for the full remainder of life. According to the recently published Life Tables for Canada, 2009-2011, the remaining life expectancy for a 45-year-old male is 36.17 years or to age 81.17. One possible calculation approach would be to determine the present value based on a fixed amount of $1,000 annually for 36.17 years. However, it can be demonstrated mathematically that this calculation will always overvalue the correct amount required. While this overstatement may initially appeal to a plaintiff's lawyer, it won't seem as attractive when the expert's calculation is discredited in court.

Another incorrect calculation approach which we sometimes see in lifetime loss valuations proceeds by discounting each future year's loss by the likelihood of survival to that year, stopping the calculation at the age at which the assumed life expectancy is reached. It's easy to show that this approach will always undervalue the loss because it "double counts" the mortality contingency discount.

> ... [W]hile life expectancy is convenient for visualization and comparison purposes, it cannot be used directly for present value calculations.
>
> **Jay Jeffery and Kelley McKeating**
> Dilkes, Jeffery & Associates

An additional misconception we occasionally encounter is that the life expectancy at birth is a correct assumption for the expected average age at death, regardless of the individual's current actual age. But clearly, if you have made it to (for example) age 70 by successfully avoiding the risks of an earlier demise, then the likelihood of a shorter lifetime must be dropped out of the average calculation. According to the above table, the remaining life expectancy for a 70-year-old male is 15.13 years, or to age 85.13, which compares to age 79.33 for a male at birth and age 81.17 for our 45-year-old male. The fallacy in this misconception is readily observed by considering the remaining life expectancy for a reasonably healthy 90-year-old female. Already dead? Not! Her remaining life expectancy is 5.35 years according to the above table.

So, while life expectancy is convenient for visualization and comparison purposes, it cannot be used directly for present value calculations. You should ensure that your expert valuator is using the correct actuarial present value calculation method.

What about reduced life expectancy calculations?

The first point to note is that while actuaries have some training in the medical underwriting field, they are not doctors and they do not have readily available mortality tables for any given medical condition. Furthermore, most attending physicians are not experienced in medical underwriting and are (properly) reluctant to opine on the remaining life expectancy of their patient.

The court-accepted proper process for establishing a reduced life expectancy (or more properly, an increased mortality risk) is to obtain an expert opinion from a qualified medical professional with experience in medical underwriting. Most commonly, this opinion will be expressed in terms of an adjustment which should be applied to the age-by-age mortality rates from the relevant table, e.g. 400 per cent (sometimes expressed as +300 per cent), or four times the standard age-by-age mortality rates of the table. This assumption will be used by your actuary to determine the loss using the actuarial present value method, and your actuary can also quote the adjusted remaining life expectancy to assist you in visualizing the effect of the medical opinion. But remember, the adjusted life expectancy cannot be used directly to perform the calculations.

In summary, the personal injury lawyer needn't be an expert in the mathematics of mortality (a field known as life contingencies). However, you'll want to ensure that your economic loss valuator is an expert in this facet of valuation — particularly if the plaintiff is not in good health.

Jay Jeffery has been an actuary since 1973 and Kelley McKeating became an actuary in 1995. Dilkes, Jeffery & Associates (www.dilkesjeffery.com) is a consulting firm that specializes in providing actuarial expert evidence services in personal injury, fatality, wrongful dismissal and other civil litigation matters.